EXPERIMENTAL STUDY ON CRITICAL RESONANT STATE OF UPSTREAM-ADVANCING WAVES

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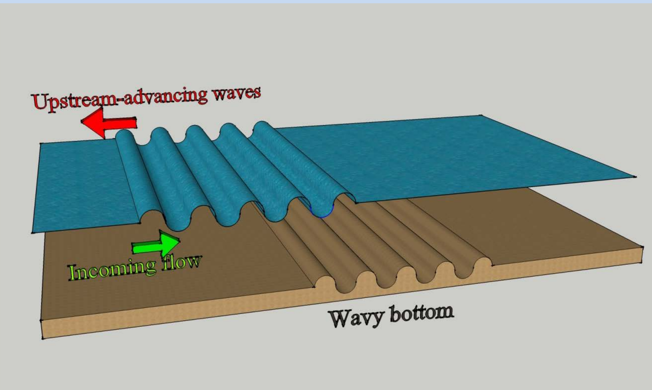
INTRODUCTION



The problem of the interaction among surface waves, current and uneven bottoms is interesting and meaningful due to a steady fluid flow traveling over periodic wavy bottoms. In coastal regions, this situation may occur when the tidal or river currents passing through the corrugated topography (particularly the periodic sandbars or periodic artificial submerged structures) in shallow water areas such as river inlets and littoral zones. Upstream-advancing waves are excited by steady flow over series fixed sinusoidal beds with specific range of the flow velocity, water depth and bottom steepness. The generation of these waves shows the state of instability and the resonant interaction relationship among surface waves, steady flow and periodic wavy bottoms. Furthermore, the critical flow velocity and critical relative water depth which stimulate the maximum wave height of upstream-advancing waves were observed through the modified physical experiment. These precise critical values will provide more accurate evidence to reflect the resonant interactive conditions and the most unstable state.

BACKGROUND

STATIONANY WAVES

Linear solution of the stationary wave profiles by flow over sinusoidal bed extending to both far upstream and downstream was presented firstly by Lamb (1932). The profile of the free surface elevation η is given by the equation (1), in which, k is the wavenumber of sinusoidal bed, b is the wave amplitude of sinusoidal bed, h is the mean water depth ($b \ll h$) and U is the steady flow velocity.

$$\eta = \frac{kb\cos kx}{k\cosh kh - (g/U^2)\sinh kh} \qquad (1)$$

The profile expression shows that it will exist a critical speed U_c as the equation (2) at which the amplitude of surface elevation becomes unbound or infinite.

$$U_c = \sqrt{\frac{g}{k}\tanh kh} \qquad (2)$$

Besides, the relationship between free water surface profile and wavy bottom profile is out of phase if $U \le U_c$ and in phase if $U \ge U_c$.

It should be noted that the waves mentioned above are stationary wave profiles which do not propagate upstream or downstream. Obviously, the infinite amplitude at critical current spend U_c cannot exist due to the nonlinearity and viscous damping under nature conditions. After that, Mei (1969) made a nonlinear analysis and obtained the steady states for the resonant case. It shows that the amplitude of the free surface is finite at the resonant velocity and the free surface amplitude can be triple-valued near the critical speed.

INSTABILITY BY RESONANCE

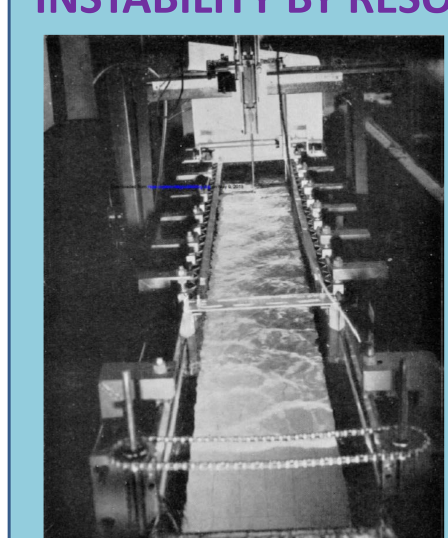


Figure 1-Binnie's experiment in 1960

Binnie (1960) observed the self-induced waves by steady flow through an open channel with vertical corrugated sides. In his experiment, continuous trains of waves were formed and moving steadily upstream on the free surface.

Yih (1976) made an analytical study about the stability of the stationary waves that form on the surface of a liquid layer flowing over a wavy bottom. Then the instability was found to be caused by resonance between primary stationary waves and a pair of disturbances which were met with Hasselmann's conditions.

$$k_3 = k_1 + k_2 \qquad (3)$$
$$\sigma_3 = \sigma_1 + \sigma_2$$

Kyotoh (1997) firstly studied the upstream-advancing waves by open channel flow over a fixed sinusoidal bed. The waves are generated with large amplitude of bottom corrugations when the Froude number is less than the resonant value as equation (2). He studied the celerity, the wavelength and the domain of existence of excited upstream-advancing waves of the flow over a sinusoidal bed. The wavelengths observed were 10-20cm, nearly 3-6 times as long as that of the bed surface. The amplitudes are just 1-2mm only meeting the judgments of existence. Then the stability analyses was made by the assumption that the upstream-advancing waves were induced by the Benjamin-Feir-type instability.

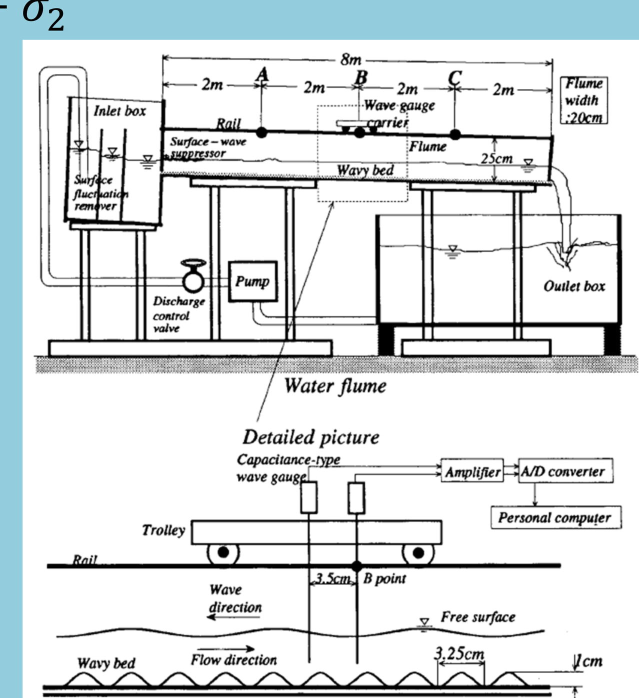


Figure 2-Kyotoh's experiment apparatus

However, only a few flow velocities was carried out by Kyotoh because he mainly focused his attention on the influence of the variation of water depth. The reason was that flow was generated by inertia with the declining slope of the flume so that the flow velocity was adjusted by changing the slope. Furthermore, the variation of the amplitude of upstream-advancing waves with the change of current velocity and relative water depth has not been observed due to the limit of his experimental apparatuses' scales.

Therefore, the domain of existence of upstream-advancing waves is too large to discover the exact resonant conditions, so that the amplitude distribution is significant for investigating the mechanism of current-topography resonant interaction in which its critical situation will excite the maximum upstream influence.

EXPERIMENTAL APPROACH

Figure 3 - experimental flume

The experiment has been carried out in a large scale wind-wave-current flume. The flume is 80m long, 1.0m wide and 1.5m deep along with eight fixed standard wooden sinusoidal bottom corrugations which are of 24cm wavelength and 8cm wave height.

The flow was generated by bump and its velocities were adjusted with an interval of 1 cm/s. The water depth relative to the bottom wavelength is varying from 0.6 to 1.4. Then the water wave profiles were measured by 14 capacitive wave gauges along the length of the flume.

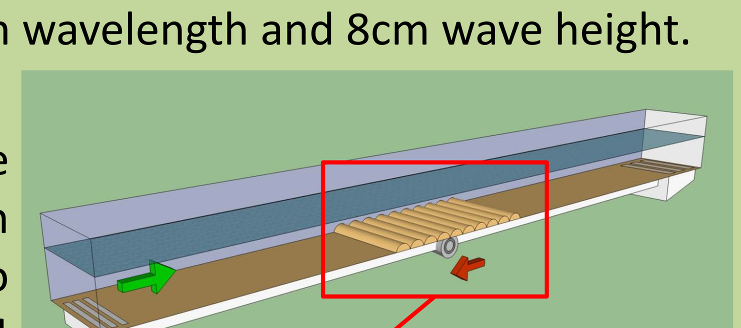


Figure 4 - schematic view of the experiment

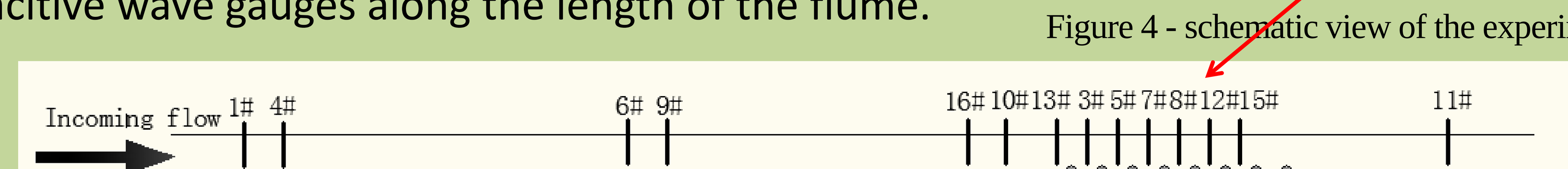


Figure 5 – Wave gauge settings

The experiment was divided into several groups. In each group, the water depth was fixed and the flow velocity was increased from low values with small increment intervals. Then the water depth was adjusted in different groups. It should be noted that the steepness of the sinusoidal wavy bottom is 0.33 (the maximum slope is 1.05 correspondingly) which is consistent with topographic setting of Kyotoh.

Group No.	Average water depth above the bottom (cm)	The ratio of water depth to bottom wavelength	Velocity range cm/s (with interval 1cm/s)
1	14.4	0.6	25-42
2	16.8	0.7	20-38
3	19.2	0.8	27-41
4	21.6	0.9	28-42
5	24.0	1.0	28-40
6	26.4	1.1	29-45
7	28.8	1.2	31-43
8	33.6	1.4	30-45

Figure 6 –Arrangement of water depths and flow velocities

KEY RESULTS

Figure 7 – Upstream-advancing waves observed by experiment

The Free-surface waves were generated and propagating upstream with a small range of current velocities along with the strong oscillation of the water body above the wavy bottom.

I. The distribution of the wave amplitudes with the change of flow velocity

The amplitudes of upstream-advancing waves surge to a maximum value rapidly at a critical flow velocity and decline sharply during the growth of the flow velocity (subcritical). The wave gauges also measured small wave components with the same frequency of upstream-advancing waves in the downstream area.

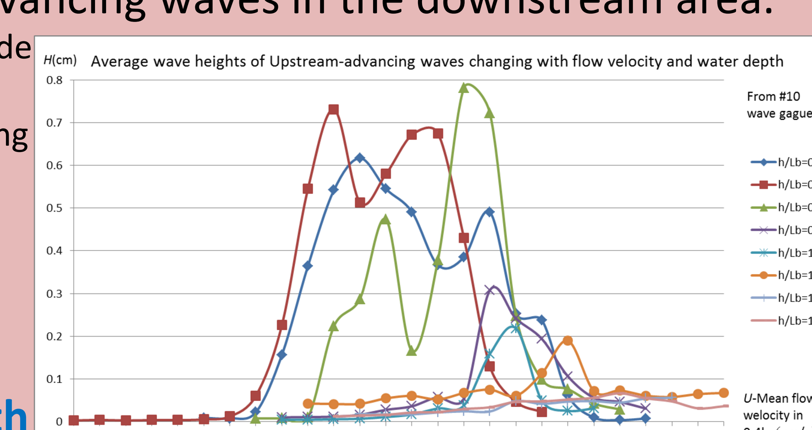


Figure 9 – Amplitude distribution of the upstream-advancing waves

II. The influence of the variation of water depth

- The upstream-advancing waves will not be generated when the ratio of water depth to bottom wavelength is larger than 1.4 .
- The maximum amplitudes from various water depths correspondingly rise to a peak value and drop down with the increase of the water depth. Besides, the current velocities which excite the maximum upstream-advancing waves shift towards larger magnitudes as the water depth increases.

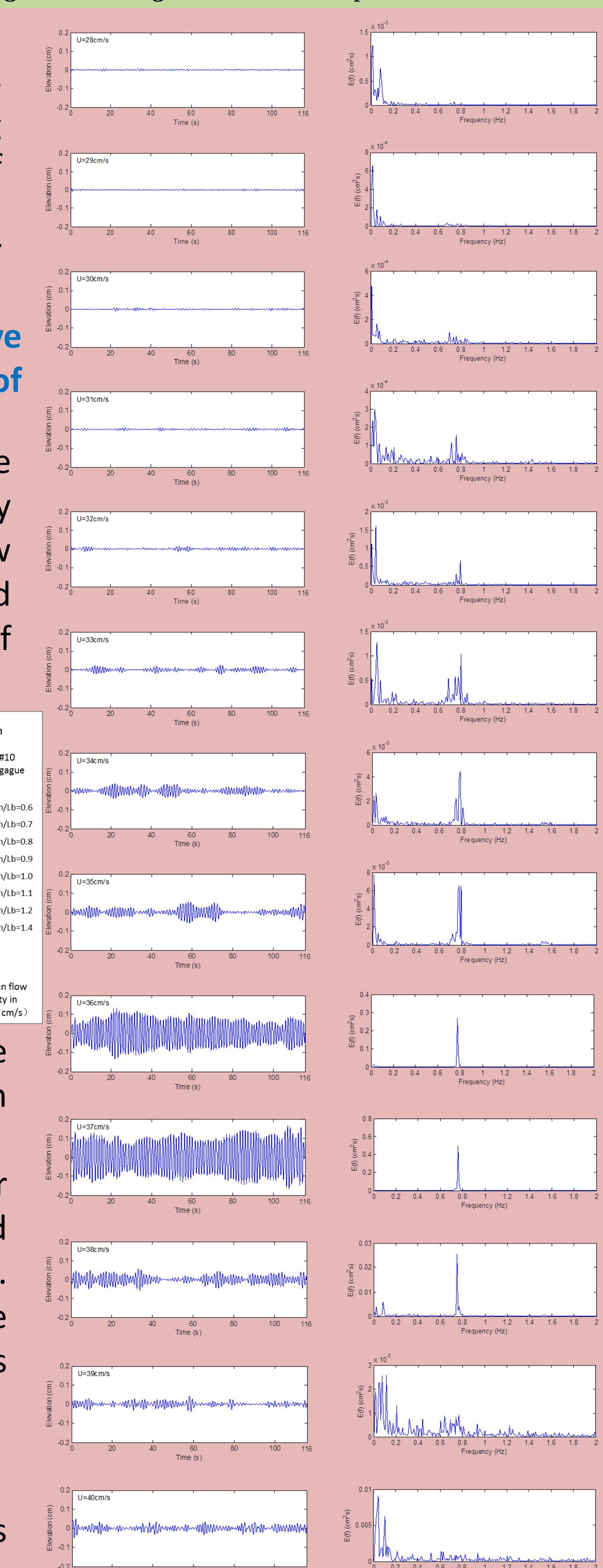


Figure 8–Free surface elevation and energy spectrum with the increment of flow velocity from Group 5

III. The period and existence of upstream-advancing waves

- The existence of upstream-advancing waves is smaller than the region from Kyotoh (Fig.10).
- The periods of the waves are concentrated between 1.2s-1.5s (Fig.11).

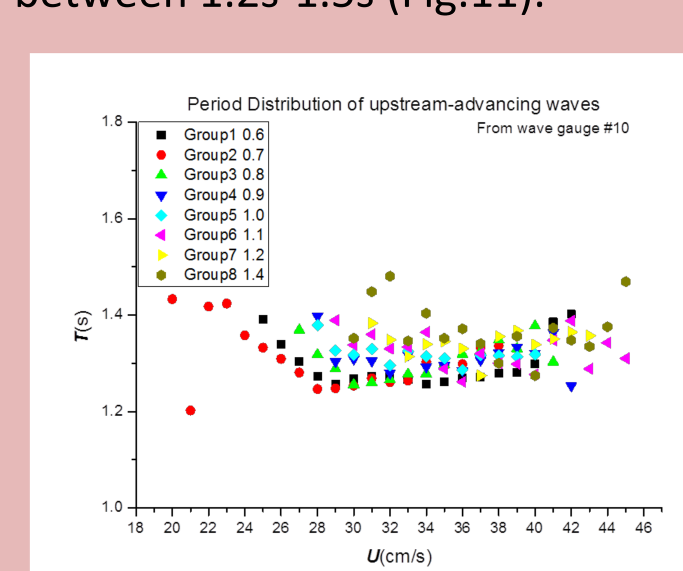


Figure 11– Period distribution of upstream-advancing waves

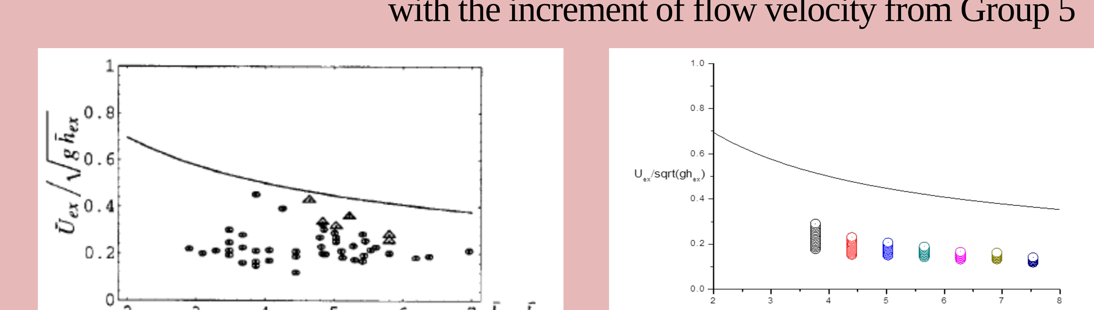


Figure 10– Comparison of the existence (Left: Kyotoh ; Right: present experiment)

In summary, the generation of upstream-advancing reflects the resonant interaction between the primary stationary waves and free surface disturbance wave components. The detailed features of this resonant type as well as the mechanism of this instability process are worthy of further study.

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